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SOLITON SOLUTIONS IN RELATIVISTIC FIELD THEORIES AND GRAVITATION

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Abstract. We report on some recent results on a class of relativistic lagrangian field theories supporting non-topological soliton solutions and their applications in the contexts of Gravitation and Cosmology. We analyze one and many-components scalar fields and gauge fields.

1 Introduction

The well-known Born-Infeld (BI) model (Born & Infeld 1934) was historically proposed to remove the divergence of the electron's self-energy in classical electrodynamics. Aside from electrostatic soliton solutions in three space dimensions, the model exhibits dyon and bidyon-like solutions (Chernitskii 1999), electric-magnetic duality (Gibbons & Rasheed 1995) and special properties of wave propagation, belonging to the class of "completely exceptional" theories (Boillat 1970).

Recently there has been a renewed interest in BI theory and its non-abelian extensions since they appear at different levels of string theory (Gibbons 1998). However, there are many other physical contexts where BI-like models have been used, such as the description of dark energy in Cosmology (e.g. Füzfa & Alimi 2006) and the phenomenological description of the nucleon structure (Deser et al 1976; Pavlovski 2002).

On the other hand, many studies on generalized electromagnetic and gauge field theories coupled with gravitation have been devoted to the analysis of particle-like solutions. Although several theorems of the 70's (Coleman 1977) forbid the existence of static, finite-energy solutions of the pure Yang-Mills theory, such solutions were found in the Einstein-Yang-Mills system (Bartnick & McKinnon 1988). Later it was shown that similar glueball solutions also exist in BI-like models in flat space (Gal'tsov & Kerner 2000) as well as in curved space (e.g. Wirschins *et al* 2001) (see Volkov & Gal'tsov (1999) for a review and references on non-abelian solitons).

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2 The models

The BI generalization of classical electrodynamics is by no means unique and, from the point of view of soliton solutions, BI theory is only a particular example of a large class, which has been exhaustively determined in Diaz-Alonso & Rubiera-Garcia (2007). Here we deal with the generalized gauge field problem, by considering gauge-invariant lagrangians which are given functions $L = \varphi(X, Y)$ of the two quadratic field invariants $X = -\frac{1}{2}\sum_a F^a_{\mu\nu}F^{\mu\nu a}$; $Y = -\frac{1}{2}\sum_a F^a_{\mu\mu}F^{*\mu\nu a}$, defined in a domain ($\Omega \subseteq \Re^2$) which is assumed to be open and connected and including the vacuum ($X = Y = 0$). In calculating the field invariants we use the ordinary definition of the trace, although other definitions are possible (Tseytlin 1997). We also require the condition $\varphi(X, Y) = \varphi(X, -Y)$ to be satisfied, in order to preserve parity invariance. Moreover we restrict our analysis to "physically admissible theories", which we define by the requirement of the vanishing of the vacuum energy ($\varphi(0, 0) = 0$), as well as the positive definiteness of the energy, which demands the minimal *necessary and sufficient* condition

$$\rho^s \geq \left(X + \sqrt{X^2 + Y^2}\right)\frac{\partial\varphi}{\partial X} + Y\frac{\partial\varphi}{\partial Y} - \varphi(X, Y) \geq 0\,, \tag{2.1}$$

to be satisfied in the entire domain of definition (Ω). The field equations read

$$\sum_b D_{ab\mu}\left[\frac{\partial\varphi}{\partial X}F^{\mu\nu b} + \frac{\partial\varphi}{\partial Y}F^{*\mu\nu b}\right] = 0\,, \tag{2.2}$$

where $D_{ab\mu} \equiv \delta_{ab}\partial_\mu + g\sum_c C_{abc}A_{c\mu}$. We next consider the electrostatic spherically symmetric solutions (ESS) ($\vec{E}_a(r) = -\vec{\nabla}(\phi_a(r)) = -\phi'_a(r)\frac{\vec{r}}{r}$; $\vec{H}_a = 0$). When this substitution is done in (2.2) we get two sets of equations

$$\nu = 0 \to \vec{\nabla}\left(\frac{\partial\varphi}{\partial X}\vec{\nabla}\phi_a(r)\right) = 0\ ;\ \nu = i = 1, 2, 3 \to \sum_{bc} C_{abc}\phi_b(r)\phi'_c(r) = 0\,. \tag{2.3}$$

The first set leads to the first integrals $r^2\frac{\partial\varphi}{\partial X}\phi'(r) = Q_a$ (Q_a being integration constants, identified as *color charges*) which coincide with the set of first integrals of the field equations for static, spherically symmetric solutions (SSS) of a multicomponent scalar field theory with a lagrangian density given by

$$L = f(\sum_a \partial_\mu\phi_a\partial^\mu\phi_a) \equiv f(X) = -\varphi(-X, Y = 0)\,. \tag{2.4}$$

The solutions of Eqs.(2.3) for the multiscalar or gauge cases take the form $\phi_a(r) = \frac{Q_a}{Q}\left(\phi(r, Q) + \alpha_a\right)$, where $Q = \sqrt{Q_a^2}$ is the mean-square (*scalar* or *color*) charge and α_a are integration constants. The function $\phi(r, Q)$ is the solution of the one-component scalar field theory resulting from the restricted lagrangian $f(X = \partial_\mu\phi\partial^\mu\phi)$. In the gauge case the second set of equations (2.3) restricts the possible

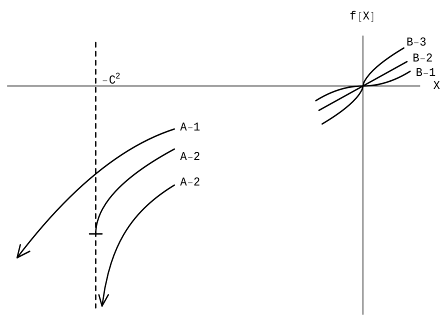

Fig. 1. Different possible central and asymptotic behaviors of the admissible models

values of the integration constants $\alpha_a = \frac{Q_a}{Q}\alpha$ and then the ESS solutions take the form $\phi_a(r) = \frac{Q_a}{Q}\left(\phi(r,Q) + \alpha\right)$ where now Q is the mean-square *color* charge.

Thus, the form of the ESS solutions of generalized gauge field theories coincides with that of the SSS solutions of an associated one-component scalar field theory. The energy of the ESS solutions, when finite, is twofold the energy of the associated SSS solutions, which reads

$$\varepsilon(Q) = -4\pi \int_0^\infty r^2 f\left(-\phi'^2(r)\right) dr = Q^{3/2}\varepsilon(Q=1). \tag{2.5}$$

The requirement of convergence of this integral leads to a classification of the admissible models according to the central (<u>A-cases</u>) and asymptotic (<u>B-cases</u>) behaviors of the soliton field strength (see Fig.1). Cases *A-1* and *A-2* correspond to infinite (but integrable) and finite soliton field strengths, respectively. Cases *B-1*, *B-2* and *B-3* correspond to asymptotic dampings of the soliton field strength which are slower than coulombian, coulombian, or faster than coulombian, respectively (Diaz-Alonso & Rubiera-Garcia 2007).

For each admissible scalar model supporting finite-energy SSS solutions there exists an infinite family of admissible gauge field theories supporting similar finite-energy ESS solutions, obtained from (2.4) and the admissibility conditions (2.1).

The linear stability of the SSS and ESS solutions requires the energy (2.5) to be a minimum against small charge-preserving perturbations of the fields and potentials in the gauge and scalar cases. In the latter we found that all finite-energy SSS solutions of admissible models are always linearly stable while in the former the following supplementary condition for stability must be satisfied

$$\frac{\partial\varphi}{\partial X} - 2X\frac{\partial^2\varphi}{\partial Y^2} > 0\,,\ \forall(X, Y=0)\,. \tag{2.6}$$

As a non-trivial example of this class of soliton-supporting theories we introduce here the family of generalized electromagnetic field models

$$\varphi(X,Y) = X/2 + \lambda X^a + \beta Y^2 (a > 3/2\,,\ \beta > 0), \tag{2.7}$$

which is the simplest generalization of a family of scalar models (defined from (2.7) by setting $\beta = 0$) supporting SSS soliton solutions, which has been analyzed in Diaz-Alonso & Rubiera-Garcia (2007). This generalized electromagnetic family fulfills the admissibility conditions (2.1) and supports ESS solitons. For certain values of the parameters in the electromagnetic case ($\lambda < 0$, $a = 2$) the non-linear term has the form of the decoupled part of the four-vertex contribution to the effective lagrangian of quantum electrodynamics.

3 Conclusions and outlook

In summary, we have analyzed scalar and generalized gauge field theories supporting soliton solutions. Aside from BI-like models which have been widely used during the last few years, other models have also recently attracted attention in search of self-gravitating scalar, electromagnetic and gauge field solitons as well as regular charged solutions (Ayon-Beato & Garcia 1999). The class of models considered here can also be useful in other physical contexts. For instance, in the extension of the description of dark energy in Cosmology, as a non-canonical scalar field (Armendariz-Picon & Lim 2005) or as a gauge field governed by generalized actions (Dyadichev et al 2002), in the phenomenological description of nucleon structure, in generalized gauge theories in higher dimensions, glueballs, etc.

References

Armendariz-Picon, C. & Lim, E.A. 2005, JCAP, 0508, 007.

Ayon-Beato, E. & Garcia, A. 1999, Gen. Rel. Grav., 31, 629.

Bartnick, R. & McKinnon, J. 1988, Phys. Rev. Lett., 61, 141.

Boillat, G. 1970, J. Math. Phys., 11, 941.

Born, M. & Infeld, L. 1934, Proc. R. Soc. London. A, 144, 425.

Chernitskii, A.A. 1999, J. High. Ener. Phys., 9912, 010.

Coleman, S. 1977, Comm. Math. Phys., 55, 113.

Deser, S., Duff, M.J., & Isham, C.J. 1976, Nucl. Phys. B, 114, 29

Diaz-Alonso, J. & Rubiera-Garcia, D. 2007, Phys. Lett. B, 653, 445.

Diaz-Alonso, J. & Rubiera-Garcia, D. 2007, Phys. Lett. B, 657, 257.

Diaz-Alonso, J. & Rubiera-Garcia, D. "A study on relativistic lagrangian field theories with non-topological soliton solutions", Preprint (2007). To be published.

Dyadichev, V., Gal'tsov, D., Zorin, A., & Zotov, M. 2002, Phys. Rev. D, 65, 084007.

Füzfa, A. & Alimi, J.-M. 2006, Phys. Rev. D, 73, 023520.

Gal'tsov, D. & Kerner, R. 2000, Phys. Rev. Lett., 84, 5955.

Gibbons, G.W. & Rasheed, D.A. 1995, Nucl. Phys. B, 454, 185.

Gibbons, G.W. 1998, Nucl. Phys. B, 514, 603.

Pavlovskii, O.V. 2002, Phys. Lett. B, 538, 202.

Tseytlin, A. 1997, Nucl. Phys. B, 501, 41.

Volkov, M.S. & Gal'tsov, D.V. 1999, Physics Reports, 319, 1.

Wirschins, M., Sood, A., & Kunz, J. 2001, Phys. Rev. D, 63, 084002.